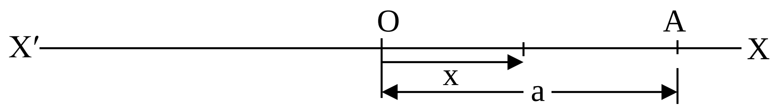
1. SIMPLE HARMONIC MOTION

A particle is said to execute simple harmonic motion if it moves such that its acceleration along its path is always directed towards a fixed point in that path, and is proportional to its displacement from the fixed point.

Let O be the fixed point on the straight line X′OX, and x the displacement of the particle from O at time t, where x is positive to the right of O.



Since the acceleration is in opposite direction to the displacement, it may be written as $-\omega^2 x$, where ω^2 is a positive constant.

Thus $\boldsymbol{\frac{d^2x}{dt^2} = -\omega^2 x}$ ……… (1)

This is the fundamental equation of simple harmonic motion. It is a differential equation, which when solved, yields a solution like

$\mathbf{x = a \sin\omega t}$ ………. (2)

From (2), the velocity, $v = a\omega\cos\omega t$

The particle has maximum velocity at point O, equal to $a\omega$.

The time taken for the particle to move from A and back to A once is called the ***period*** (T) of the oscillation. i.e. $T = \frac{2\pi}{\omega}$, where ω is the angular frequency of the oscillation.

a is the maximum displacement of the particle from the centre O and is called the ***amplitude*** of the oscillation. Thus, $v_{max} = a\omega$

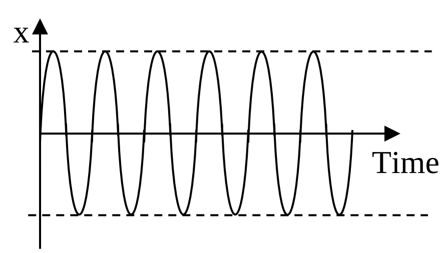
Damping

Simple harmonic motion is said to be damped if the amplitude decreases with time. Any of the following three cases is possible:

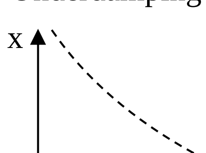
(i) Ungerdamping
(ii) Critical damping
(iii) Overdamping

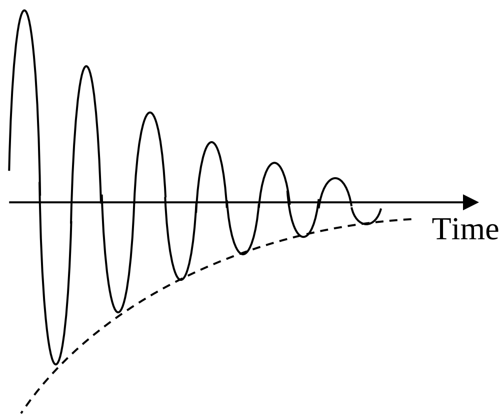
Illustrations:

(a) Free Oscillation – the amplitude is constant

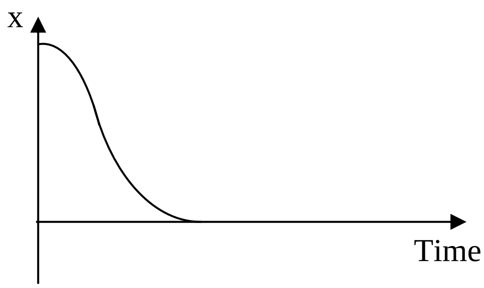


(b) Damped Oscillations
(i) Underdamping – amplitude decreases gradually

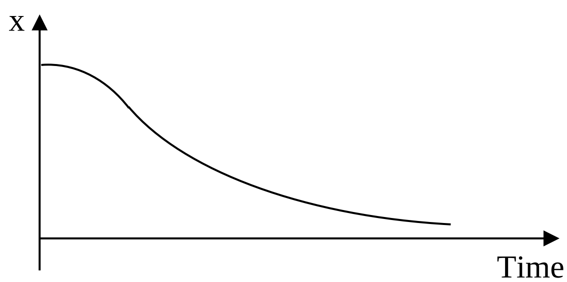




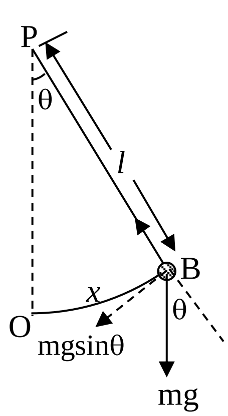
(ii) Critical Damping – the motion is stopped at the equilibrium position in the shortest possible time, e.g. in galvanometers



(iii) Overdamping - particle may not manage to return to the equilibrium position in expected normal time



The Simple Pendulum



Consider an inelastic string of length l whose end is fixed at P, and to whose other end is fixed a mass m.

Suppose the mass m is freely oscillating such that at a certain instant the length of the arc OB is x when the string makes an angle θ with the vertical. Then the force pulling m towards O along OB is mg sinθ.

Let a = acceleration of m (being positive in a direction away from O)

Then ma = -mg sinθ

But when θ is small ⇒ $\sin\theta \approx \theta = \frac{x}{l}$

Thus ma = -mgθ = $-mg\frac{x}{l}$

$$\therefore \quad \frac{-g}{l}x = -\omega^2 x, \quad where\ \omega^2 = \frac{g}{l}$$

Since the acceleration is proportional to the displacement, x, from O and the negative sign implies it is towards O, the mass executes simple harmonic motion.

The period, $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{l}{g}}$

This expression provides a way of determining g using a simple pendulum. Discuss the experiment.

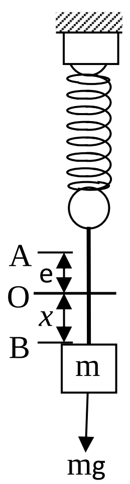
The Helical Spring

If a force is applied to a spring so that the spring extends by a distance x, the tension, T, produced in the spring is proportional to x.
i.e. $T = kx$,
where k is a constant of the spring, called the ***force constant***.

Example:

A mass, m, freely suspended on a vertical spring, causes an extension e of the spring. Show that when m is slightly displaced vertically from the equilibrium position and then released, it executes simple harmonic motion of period $T = 2\pi\sqrt{\dfrac{e}{g}}$.

Solution:



Before m is pulled, the tension in the spring is mg = ke …. (1)
Let O be the equilibrium position
Suppose at a certain instant during its motion m is at a distance x below O. Then the tension in the spring is k(e + x)
Hence ma = mg – k(e + x), where a is the acceleration (positive away from O)
Substituting for mg using (1) gives
Ma =ke – ke–kx

$$\therefore \quad a = \frac{-k}{m}x = -\omega^2 x, \qquad where\ \omega^2 = \frac{k}{m}$$

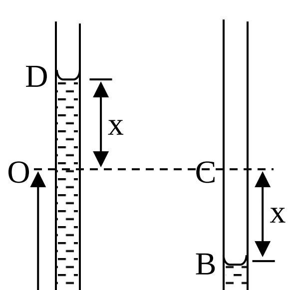
The negative sign means that the acceleration is towards O, and since it is proportional to the displacement x from O, m executes simple harmonic motion.

But since $mg = ke \Rightarrow \dfrac{m}{k} = \dfrac{e}{g}$

$$\therefore \quad T = 2\pi\sqrt{\frac{e}{g}}$$

Thus, a helical spring may be used to determine g. Discuss the procedure.

Liquid in a U-Tube



Equilibrium positions are O and C. Suppose the level on the right is displaced to B by blowing gently.
The excess pressure on the whole liquid
= excess height x density of the liquid x g

$= 2x\rho g$

Thus, the force on the liquid $= 2x\rho gA$,

where A is the cross-sectional area of the tube.

The mass of the liquid $= 2hA\rho$

The acceleration, a, of the liquid towards O or C is given by

$$-2x\rho gA = 2hA\rho a$$

$$\therefore \quad a = \frac{-g}{h}x = -\omega^2 x$$

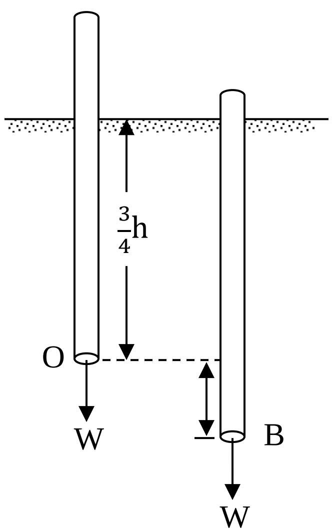
where $\omega^2 = \frac{g}{h}$, i.e the motion of the liquid about O or C is simple harmonic.

$$\text{The period, } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{h}{g}}$$

Example:

A uniform cylinder of height h floats vertically in water with ¾ of its height submerged. Show that if the cylinder is given a small vertical displacement and then released, it executes simple harmonic motion of period $\pi\sqrt{\frac{3h}{g}}$

Solution:



When freely floating, the height submerged is $\frac{3}{4}h$

Weight of the cylinder, W = weight of water displaced

$= \frac{3}{4}hA\rho g$

where ρ is the density of water.

During motion, suppose at a certain instant the lower end is at a distance x below the equilibrium, O.

Then, the upthrust, $U = (\frac{3}{4}h + x)A\rho g$

Let a = acceleration (positive away from O)

Then, using $ma = W - U$, where $m = \frac{3}{4}hA\rho$

$ma = \frac{3}{4}hA\rho g - \frac{3}{4}hA\rho g - xA\rho g$

$\therefore \frac{3}{4}hA\rho a = -xA\rho g$

$$\therefore \quad a = \frac{-4g}{3h}x$$

The negative sign means that the acceleration is towards O, and since it is proportional to the displacement x from O, the cylinder executes simple harmonic motion.

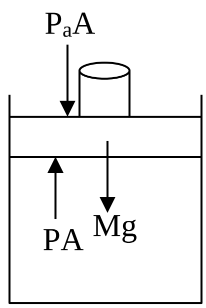
$$\text{Now} \quad a = -\omega^2 x, \qquad \text{where } \omega^2 = \frac{4g}{3h}$$

$$\therefore \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{3h}{4g}} = \pi\sqrt{\frac{3h}{g}}$$

Example:

A cylindrical vessel of cross-sectional area A, contains air of volume V at a pressure P, trapped by a frictionless piston of mass M. The piston is pushed down and released. Show that the frequency of oscillation is f = $\frac{A}{2\pi}\sqrt{\frac{P}{MV}}$

Solution:



Let P_a = atmospheric pressure

Before the piston is pushed, $P_aA + Mg = PA$

When the piston is pushed in, suppose at a certain instant it is at a distance x below the equilibrium position. Then its pressure is $(P + \delta P)$, while its volume is $(V - \delta V)$

Using PV = constant, we have

$$PV = (P + \delta P)(V - \delta V)$$

$$\therefore PV = PV + V\delta P - P\delta V - \delta P\delta V$$

Ignoring $\delta P\delta V$

$$V\delta P = P\delta V$$

$$\therefore \quad \delta P = \frac{P}{V}\delta V$$

Now, if a is the acceleration (positive downwards)

$$(P_aA + Mg) - (P + \delta P)A = Ma$$

$$PA - PA - \delta PA = Ma$$

$$\therefore \quad a = \frac{-\delta PA}{M} = \frac{PA\delta V}{MV}$$

But $\delta V = Ax$

$$\therefore \quad a = \frac{-PA^2x}{MV}$$

$$\therefore \quad 4\pi^2f^2 = \frac{PA^2}{MV}$$

$$\therefore \quad f = \frac{A}{2\pi}\sqrt{\frac{P}{MV}}$$

P.E and K.E Exchanges in Oscillating Systemsb

The energy of a stretched spring is potential energy (p.e).

For an extension x,

$$\text{p.e} = \int Fdx = \int kxdx = \tfrac{1}{2}kx^2$$

The energy of the mass is k.e = $\frac{1}{2}mv^2$, where v is the velocity.

Now, $x = a\sin\omega t$

$\therefore v = \omega a\cos\omega t$

The total energy of the spring plus mass = $\frac{1}{2}kx^2 + \frac{1}{2}mv^2$

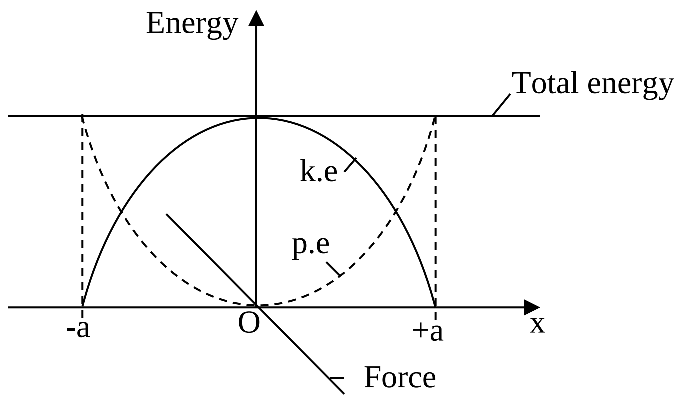
$$= \tfrac{1}{2}ka^2\sin^2\omega t + \tfrac{1}{2}m\omega^2a^2\cos^2\omega t$$

But $k = m\omega^2$

$\therefore$ Total energy $= \frac{1}{2}m\omega^2a^2(\sin^2\omega t + \cos^2\omega t)$

$= \frac{1}{2} m\omega^2 a^2 = \text{constant}$

Thus the total energy of the system of vibrating mass and spring is constant.
i.e. p.e = 0 when k.e is maximum and vice versa. The diagram below shows the variation of both p.e and k.e with displacement of an oscillating system.



UNEB 2012 No 2

a) Define the following terms as applied to oscillating motion

i) Amplitude [1mk]

ii) Period [1mk]

b) State four characteristics of simple harmonic motion [2mk]

A mass m, is suspended from a rigid support by a string of length, l. the mass is pulled a side so that the string makes an angle, θ with the vertical and then released.

i) Show that the mass executes simple harmonic motion with a period, √ – [05mk]

ii) Explain why this mass comes to a stop [02mk]

A piston in a car engine performs simple harmonic motion of frequency 12.5Hz. If the mass of the piston is 0.50kg and its amplitude of vibration is 45mm, find the maximum force on the piston. **An[139N]** [03mk]

e) Describe an experiment to determine the acceleration due to gravity, g using a spiral spring of known force constant [06mk]

UNEB 2011 No 2

a) i) what is meant by simple harmonic motion [1mk]

ii) State two practical examples of simple harmonic motion [1mk]

iii) Using graphical illustration distinguish between under damped and critically damped oscillation [4mk]

b) i)describe an experiment to measure acceleration due to gravity using a spiral spring [6mk]

ii) State two limitations to the accuracy of the value it b (i) [02mk]

UNEB 2010 No 2

b) i) What is meant by a simple harmonic motion [1mk]

ii) Distinguish between damped and forced oscillations [2mk]

c) a cylinder of length , cross sectional area A and density, floats in a liquid of density, , the cylinder is pushed down slightly and released.

i) Show that a performs simple harmonic oscillation [5mk]

ii) Derive the expression for the period of oscillation [2mk]

An(T= √(

d) A spring of force constant 40Nm^{-1} is suspended vertically. A man of 0.1kg suspended from the spring is pulled down a distance of 5mm and released. Find the,

i) Period of oscillation **An[0.314s]** [2mk]

ii) Maximum oscillation of the mass **An[2ms^{-2}]** [2mk]

iii) Net force acting on the mass when it is 2mm below the centre of oscillation. **An[0.08N]** [2mk]

UNEB 2009 No 3

(a) What is meant by simple harmonic motion (01marks)

A cylindrical vessel of cross-sectional area A, contains air of volume V, at a pressure P, trapped by frictionless air tight piston of mass M,. The piston is pushed down and released.

(i) If the piston oscillates with s.h.m, show that the frequency is given by — √ —

(06marks)

(ii) Show that the expression for, f in b(i) is dimensionally correct (02marks)

(c) Particle executing s.h.m vibrates in a straight line, given that the speeds of the particle are 4m and 2m when the particle is 3cm and 6cm respectively from equilibrium. calculate the;

(i) amplitude of oscillation **An(6.7x m)** (03marks)

(ii) frequency of the particle **An(10.68Hz)** (03marks)

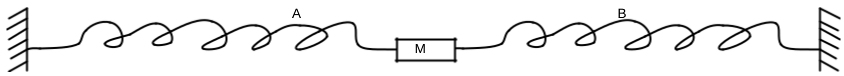
Give two examples of oscillatory motions which execute s.h.m and state the assumptions made in each case

UNEB 2008 No3

a) (i) Define simple harmonic motion [01marks]

(ii) A particle of mass m executes simple harmonic between two point A and B about equilibrium position O. Sketch a graph of the restoring force acting on the particle as a function of distance r and moved by the particle [02marks]

b)



Two springs A and B of spring constants K_A and K_B respectively are connected to a mass m as shown. The surface on which the mass slides is frictionless.

Show that when the mass is displaced slightly, it oscillates with simple harmonic motion of frequency given by

— √(——) [04marks]

(ii) If the two springs above are identical such that 5Nm^{-1} and mass m=50g, calculate the period of oscillation **An[0.44s]** [03marks]

UNEB 2007 No 1

a) Define simple harmonic motion [01marks]

b) Sketch a graph of

i) velocity against displacement [03marks]

acceleration against displacement for a body executing S.H.M

A glass U-tube containing a liquid is tilted slightly and then released

i) Show that the liquid oscillates with S.H.M [04marks]

ii) Explain why the oscillations ultimately come to rest [03marks]

UNEB 2007 No 4

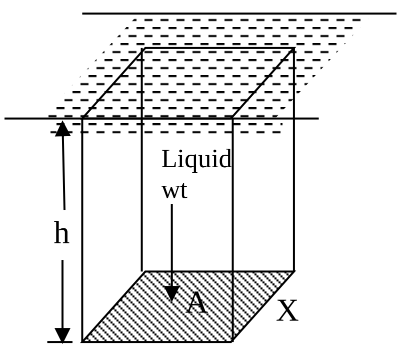
b) i)What is meant by acceleration due to gravity.

ii)Describe how you would use a spiral string, a retort stand with a clamp, a pointer, seven 50g masses, meter rule and a stop clock to determine the acceleration due to gravity [6mk]

iii) State any two sources of errors in the experiment in bii) above. [01mark] iv)A body of mass 1kg moving with simple harmonic motion has speed of 5ms^{-1} and 3ms^{-1} when it is at a distance of 0.1m and 0.2m respectively from the equilibrium point. Find the amplitude of motion [04marsk]

2. FLUIDS

Formula for Pressure



Suppose a horizontal plate X of area A is placed at a depth h below the liquid surface. The force on X due to the liquid is equal to the weight h and uniform cross-sectional A.

i.e.weight = Ahρg,

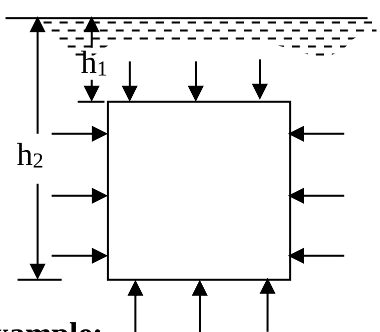
where g is the gravitational acceleration and ρ is the density of the liquid.

$$\therefore \quad \text{pressure, P, on X} = \frac{\text{force}}{\text{area}} = \frac{\text{Ah}\rho\text{g}}{\text{A}} = \mathbf{h\rho g}$$

Archimedes' Principle

When a body is wholly or partially immersed in a fluid it experiences an upthrust equal to the weight of the fluid displaced.

This may be demonstrated as follows:



Imagine a solid M immersed in a fluid of density ρ

Then,

(i) the resultant horizontal force is zero.

(ii) the resultant upward force on the solid is

$h_2\rho gA - h_1\rho gA = (h_2 - h_1)\rho gA$

But $(h_2 - h_1)A$ = volume of the solid

$\therefore$ $(h_2 - h_1)\rho gA$ = weight of the fluid displaced by the solid.

Hence, upthrust = weight of the fluid displaced.

Example:

Derive an expression for the upthrust experienced by a body of mass M and density d_1 when it is totally immersed in a fluid of density d_2.

Solution:

Volume of body $= \dfrac{M}{d_1}$ = volume of displaced fluid

Thus, mass of the displaced fluid = $M\dfrac{d_1}{d_2}$

The upthrust on the body = weight of the fluid displaced = $\frac{d_1}{d_2}$Mgx of the stem submerged.

Flotation

Law:

A floating body displaces its own weight of the fluid in which it floats.

Example:

A hydrometer has a bulb of volume and a stem of cross-section a. It floats in a liquid of density d_1 with a length submerged. Show that when it floats in a liquid of density d_2, where $d_1 > d_2$, the stem sinks a further distance

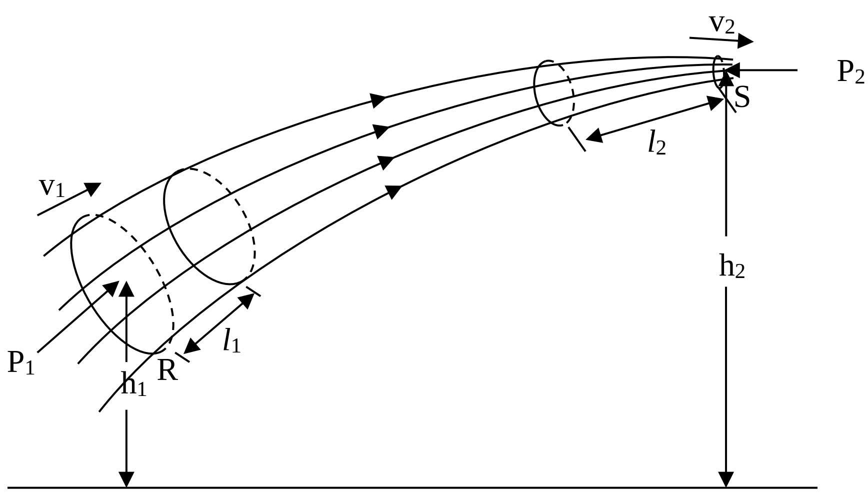
$$\frac{(V + ax)(d_1 - d_2)}{ad_2}$$

What happens if $d_1 < d_2$?

FLUIDS IN MOTION

Pressure and Velocity

There is a relationship between the pressure and velocity at different parts of a moving incompressible fluid. Imagine such a fluid flowing as the streamlines below show.



Let P_1 be the pressure at R and P_2 the pressure at S.

If the fluid is non-viscous, the work done by the pressure difference, $(P_1 - P_2)$, per unit volume of a fluid flowing along a pipe steadily, is equal to the gain in kinetic energy per unit volume plus the gain in potential energy per unit volume.

Now, the work done per unit volume at R is equal to P_1.

and the work done per unit volume at S is equal to P_2.

$\therefore$ net work done on the fluid per unit volume is $P_1 - P_2$

The kinetic energy per unit volume $= \frac{1}{2}\rho v^2$, where ρ = density of the fluid

$\therefore$ kinetic energy gained per unit volume $= \frac{1}{2}\rho(v_2^2 - v_1^2)$

and potential energy gained per unit volume $= \rho g(h_2 - h_1)$

So, as stated above

$$P_1 - P_2 = \tfrac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(h_2 - h_1)$$

$$\therefore \quad P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2$$

$$= P + \tfrac{1}{2}\rho v^2 + \rho g h$$

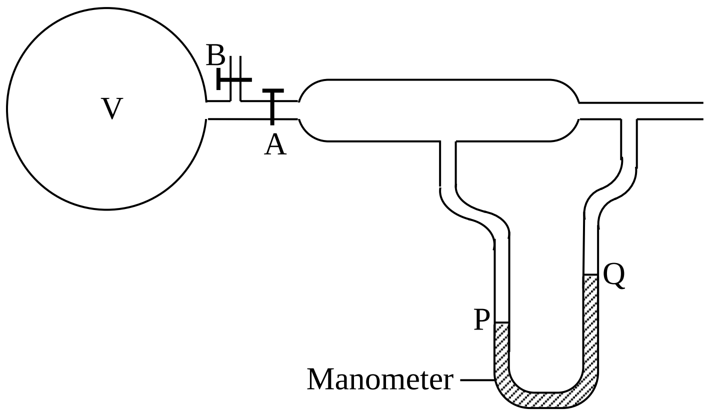
$$= \text{constant}$$

Thus

The sum of the pressure at any part, plus the kinetic energy per unit volume, plus the potential energy per unit volume there is always constant.

This is known as ***Bernoulli's principle***.

The following arrangement can be used to verify Bernoulli's principle.

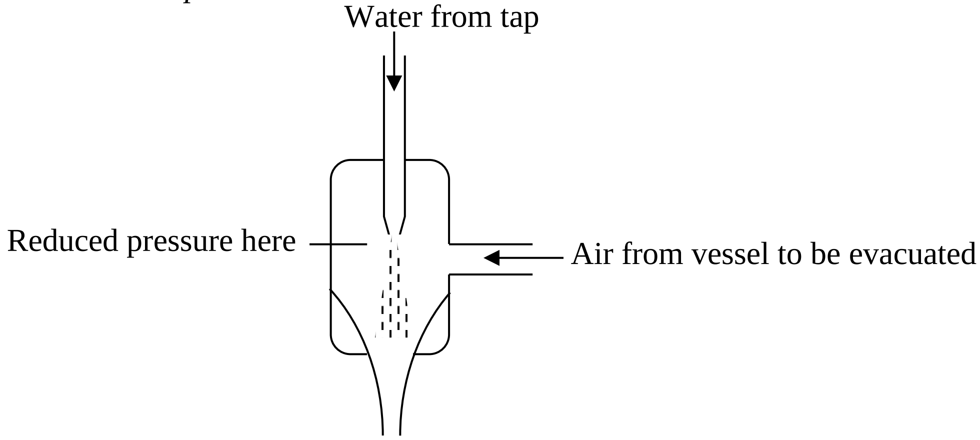


- The apparatus is set up as shown, with the manometer having a suitable liquid.
- Tap A is closed while B is opened and air is pumped into vessel V.
- Tap B is then closed and A is opened and the liquid levels in the manometer are observed.

The observations indicate that the pressure in the narrower part of the tube, where the velocity is higher, is lower.

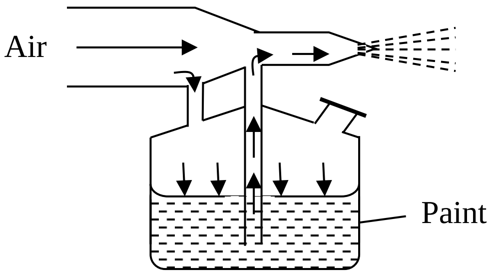
Applications of Bernoulli's Principle

1. *The Filter Pump*

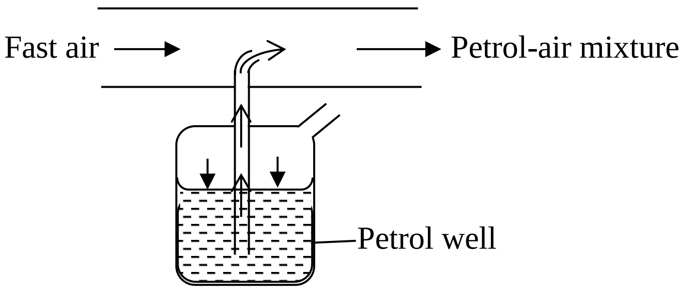


The water from the tap jets out of the tube at high speed and this makes the air in the region of the jet to move faster than that elsewhere. This, according to Bernoulli's principle, leads to reduced pressure in that region. So air is sucked from the vessel connected to the filter pump and is carried along with the flowing water.

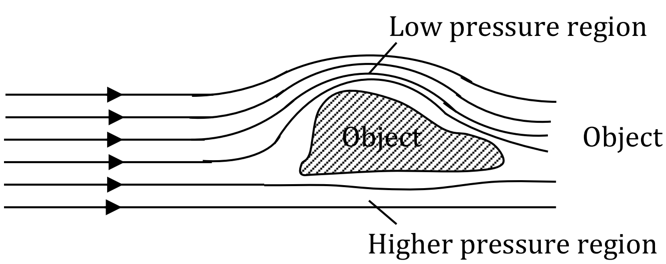
2. *Paint Spray*



3. *Carburetor*



4. *Aero foil Lift*



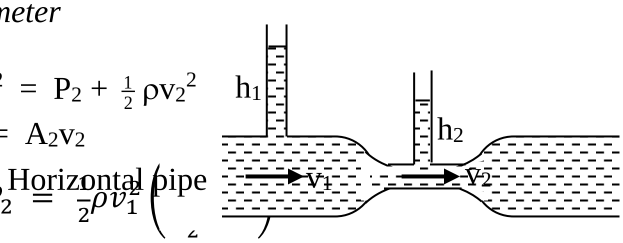
Flow Meters

1. *Venturimeter*

$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

But $A_1 v_1 = A_2 v_2$

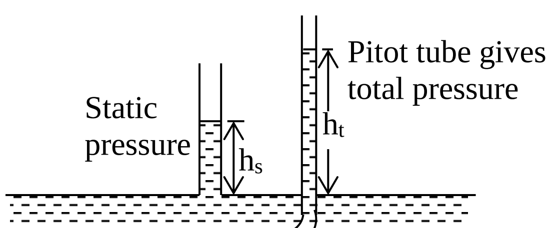
$\therefore \quad P_1 - P_2 = \frac{1}{2}\rho v_1^2 ($



So v_1 can be found.

However, this expression is valid for only streamline flow of non-viscous incompressible fluids. So it cannot be applied to gases, heavy oils and very rapid flow.

The PitotTube



Total pressure = P + $\frac{1}{2}\rho v^2$
Static pressure = P
Dynamic pressure = $\frac{1}{2}\rho v^2$

$$\therefore \quad v = \sqrt{\frac{2}{\rho}(total - static)} = \sqrt{\frac{2}{\rho}(h_t - h_s)}$$

Example:

Water flows steadily along a uniform flow tube of cross-section 30 cm^2. The stati pressure is 1.20 x 10^5 Pa and the total pressure is 1.28 x 10^5 Pa. Calculate the flow velocity and the mass of the water per second flowing past a section of the tube.
[Density of water = 1000 kg m^{-3}]

Solution:

$P_{tot} = P_{stat} + P_{dyn}$
$= P_{stat} + \frac{1}{2}\rho v^2$

$$\therefore \quad v = \sqrt{\frac{2}{\rho}(P_{tot} - P_{stat})} = \sqrt{2 \text{ x } 10^{-3}(1.28 - 1.2) \text{x } 10^5} = 4 \text{ ms}^{-1}$$

Mass per second = Avρ = 30 x 10^{-4} x 4 x 1000 = $\underline{\underline{12 \text{ kg s}^{-1}}}$

3. VISCOSITY

At low speeds of flow of a liquid, the molecules of the liquid move in layers at different speeds. Thus the molecules in the adjacent layers rub over each other. So the bulk of the fluid undergoes a shear. Because of intermolecular attraction, there is resistance to the flow.
Viscosity is the fluid's resistance to shear.

Newton's Formula

The viscous force, F $\propto$ A x velocity gradient, where A is area of the layer of the fluid
$\therefore$F =ηA x velocity gradient ……………… (1)
whereη is a constant called the ***coefficient of viscosity.***
The velocity gradientis the velocity differenceof fluid per unit distance traversed in a direction perpendicular to the flow of the fluid.
Q: List the similarities and differences between solid friction and viscous frictional force.

From equation (1) $\eta = \dfrac{F}{\text{A x velocity gradient}}$ ……(2)

Thus, ***the coefficient of viscosity is the tangential force acting on an area of 1 m^2 of fluid which resists the motion of one layer over another when the velocity gradient between them is 1 s^{-1}.***

The unit of η is $\dfrac{\mathrm{N}}{\mathrm{m^2 m s^{-1} m^{-1}}} = \mathrm{N\ s\ m^{-2}}$ (or $\mathrm{kg\ s^{-1} m^{-1}}$)

Steady Flow of Liquid through a Pipe

The volume of liquid issuing per second from the pipe depends on:
(i) the coefficient of viscosity, η
(ii) the radius of the pipe, r
(iii) the pressure gradient set up along the pipe, s

i.e. volume per second = $k\eta^x r^y s^z$
where k is a constant and x, y and z are indices to be found.
Using dimensions: $L^3T^{-1} = (ML^{-1}T^{-1})^x L^y (ML^{-2}T^{-2})^z$

$\therefore 0 = x + z$
$3 = {}^-x + y - 2z$
$1 = x + 2z$

from which $x = {}^-1$, $z = 1$, $y = 4$

$$Hence, volume\ per\ second = \frac{kr^4 s}{\eta} = \frac{k\Delta p r^4}{l\eta}$$

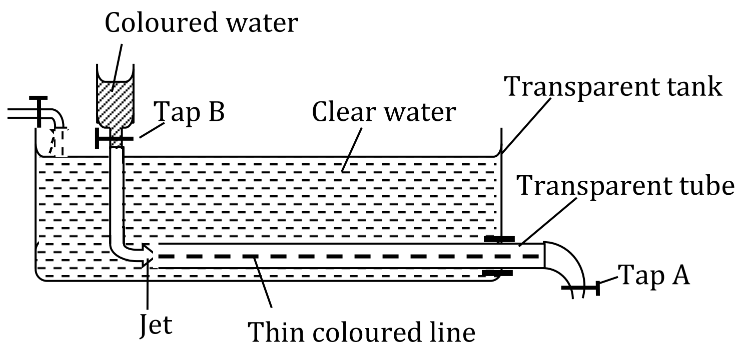
The constant $k = \dfrac{\pi}{8}$

Hence, volume per second $= \dfrac{\pi \Delta p r^4}{8 l \eta}$

This is known as ***Poiseuille's formula***. It applies only to laminar flow.

<u>Q:</u> Distinguish between laminar and turbulent flow.

Demonstration of Streamline and Turbulent Flow



- A transparent tank, fitted with a horizontal transparent tube is filled with water from a tap. Tap A controls the rate of flow through the horizontal tube while tap B opens for the coloured liquid.
- Tap A is opened, first slightly and then B is opened to release some coloured liquid.
- Tap A is progressively opened further.

Observation:

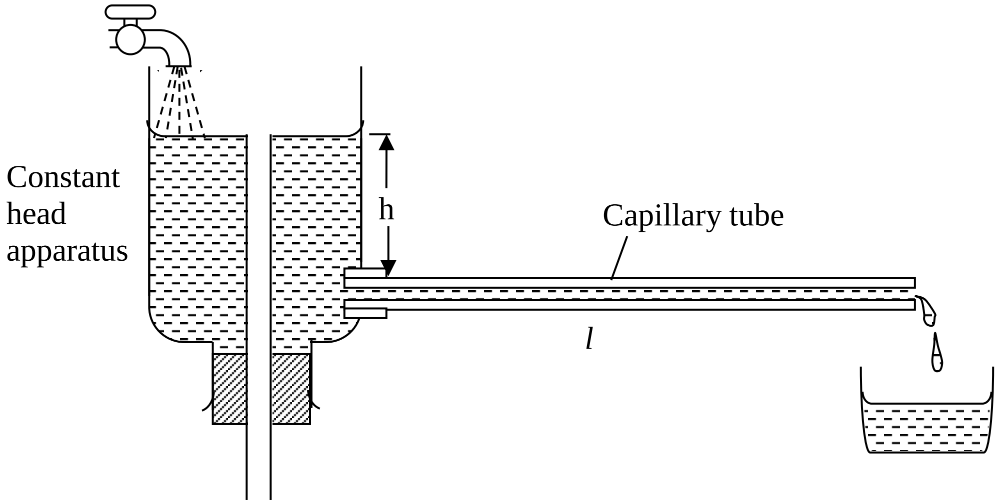
At first a thin coloured line is seen in the horizontal tube. This is streamline flow. However, as A is opened further, the coloured line disappears and instead the colour fills the whole tube. The flow has now become turbulent.

Example

Water flows steadily through a horizontal tube consisting of two parts joined end to end; one part is 21 cm long and has a diameter of 0.225 cm and the other is 7.0 cm and has a diameter of 0.075 cm. If the pressure difference between the ends of the tube is 14 cm of water, find the pressure difference between the ends of each part.

Determination of Viscosity by Poiseulli's Formula

- The length, *l*, and the radius, r, of the capillary tube are measured and noted. Then the apparatus is set up as shown.

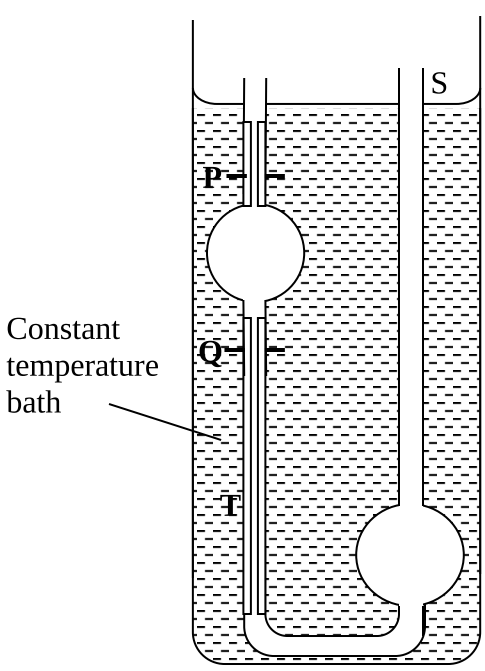


- The constant head tank is constantly filled with the liquid under test.
- The liquid is collected over a measured time interval and the volume flowing per second, V, is found.

Then $V = \dfrac{\pi h \rho g r^4}{8\eta l}$

From which $\eta = \dfrac{\pi h \rho g r^4}{8Vl}$

Comparison of Viscosity Using Ostwald Viscometer



- The first liquid is introduced at S and drawn by suction above mark P.
- The time, t_1, taken for the liquid level to fall between fixed marks P and Q is noted.
- The procedure is repeated with the same volume of the second liquid and the time, t_2, for the liquid level to fall from P to Q is noted.

Let ρ_1 and ρ_2 be the respective densities of the liquids, r the radius of tube T, *l* the length of T, h the average head of the liquid forcing through T in each case and V the volume between the marks P and Q.

Then $\dfrac{V}{t_1} = \dfrac{\pi(h\rho_1 g)r^4}{8\eta_1}$ ... ... (1)

$$\text{and} \quad \frac{V}{t_1} = \frac{\pi(h\rho_2 g)r^4}{8\eta_2} \quad \ldots\ldots \quad (2)$$

$$\therefore \quad \frac{t_1}{t_2} = \frac{\eta_1\rho_2}{\eta_2\rho_1}$$

$$\therefore \quad \frac{t_1}{t_2} = \frac{t_1}{t_2}.\frac{\rho_1}{\rho_2}$$

Ostwald's viscometer can also be used to:
(i) measure the viscosity of a liquid
(ii) investigate the variation of viscosity with temperature
Discuss the procedure of each experiment.

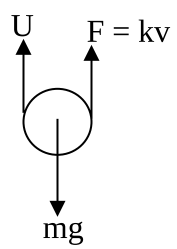
Stokes' Law

The viscous force, F, on a sphere of radius r moving at a velocity v in a liquid of viscosity η is given by

$\mathbf{F = 6\pi r\eta v}$

Terminal Velocity

Consider a sphere of mass m and radius r released to fall through a fluid of viscosity η. Then three forces act on it during its motion, i.e. its weight, mg, upthrust, U, and the viscous drag, F = kv, where $k = 6\pi r\eta$ as shown.

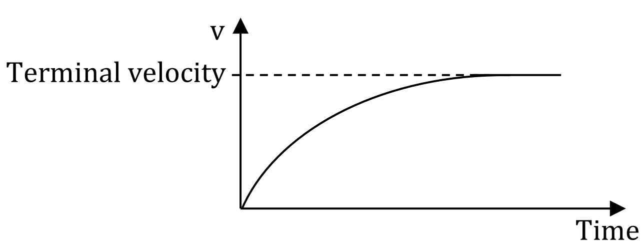


If a is the acceleration, then $ma = mg - U - kv$

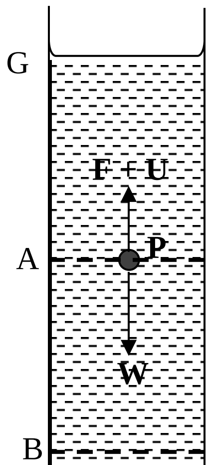
Thus the sphere accelerates at first and the acceleration goes on decreasing until it becomes zero at a maximum velocity, v_o, such that

$kv_o + U = mg$

The maximum velocity, v_o, attained is known as the ***terminal velocity*** and it is reached when the sum of the upthrust and the viscous force is equal to the weight of the body. This is so for any shape.



Comparison of Viscosities of Very Viscous Liquids



- A tall glass vessel, G, is filled with the liquid, and a small steel ball, P, is dropped gently into the liquid along the axis of G.
- When the ball has moved some distance down, its terminal velocity, v_1, is measured by timing its fall through a distance AB.

Then the upthrust, $U = \frac{4}{3}\pi\sigma g r^3$

where r = radius of the ball and σ = density of the liquid.

The weight, W of P is $\frac{4}{3}\pi\rho g r^3$, where ρ = density of P

Thus, $\quad 6\pi\eta_1 r v_1 = \frac{4}{3}\pi g r^3(\rho - \sigma_1)$

$$\eta_1 = \frac{2gr^2(\rho - \sigma_1)}{9v_1} \quad \dots\dots\dots. \quad (1)$$

The experiment is repeated with the second liquid, of viscosity η_2 and density σ_2, using the same steel ball.

Then $$\eta_2 = \frac{2gr^2(\rho - \sigma_2)}{9v_2} \quad \dots\dots\dots. \quad (2)$$

where v_2 is the new terminal velocity.

From (1)and (2) $\frac{\eta_1}{\eta_2} = \frac{v_2(\rho - \sigma_1)}{v_1(\rho - \sigma_2)}$

Viscosity in Gases

- Viscosity in gases is due to momentum transfer between the neighbouring layers of gases.
- It is proportional to the average speed of the gas molecules.
- So, increase in gas temperature increases viscosity

Examples:

1. Two identical spherical rain drops falling each with terminal velocity v_1 coalesce into a larger drop that moves with terminal velocity v_2. Find the ratio $v_1 : v_2$.

2. A sphere of radius 2 cm and mass 100 g, falling vertically through air of density 1.2 kg m^{-3}, at a place where g 9.81 m s^{-2}, attains a steady velocity of 30 m s^{-1}.
Find the value of k. **[2.3]**

3. Castor oil at 20^{o}C has a coefficient of viscosity 2.42 N s m^{-2} and a density 940 kg m^{-3}. Calculate the terminal velocity of a steel ball of radius 2 mm falling under gravity in the oil. [Density of steel is 7800 kg m^{-3}]. **[0.025 m s^{-1}]**

A metal ball of diameter 10mm is timed as it falls through oil at a steady speed, it takes 0.5s to fall through a vertical distance of 0.03m. Assuming that density of the metal is 7500kgm^{-3} and that of oil is 900kgm^{-3}, find

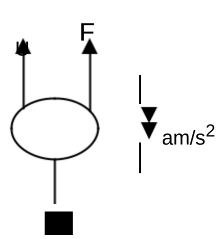
i. The weight of the ball (2 marks)

The up thrust on the ball

iii. The coefficient of viscosity of oil (03 marks (Assume the viscous force = 6π r V_0 where is the coefficient of viscosity, r is radius of *the ball and*

V_0 is terminal velocity)

Solution



= – ( )

Weight = 0.31N

ii) Up thrust U –

Weight = mg

$-$ ( )

U = 0.037N

iii) At terminal velocity Mg = U +F

0.31 = 0.037 + 6π r V_0

but V_o =

2.414Nsm^{-2}

Exercise:27

1. A small oil drop falls with terminal velocity of $4x10^{-4}ms^{-1}$ through air. Calculate the radius of the drop. What is the terminal velocity of oil drop if its radius is halved.
   (viscosity of air = $1.8x10^{-5}Nm^{-2}s$, density of oil = $900kgm^{-3}$, neglect density of air)
   An [$1.92x10^{-6}m$, $1.0x10^{-4}ms^{-1}$]

2. A spherical rain drop of radius m falls vertically in air at 20 . If the densities of air and water are 1.2 and 1000 respectively and that the coefficient of viscosity of air at 20 is s, calculate the terminal velocity of the drop. **An[4.484 m].**

A metal sphere of radius $2.0x10^{-3}m$ and mass $3.0x10^{-4}kg$ falls under gravity, central down a wide tube filled with a liquid at 35^0c, the density of the liquid is $700kgm^{-3}$, the sphere attains a terminal velocity of magnitude $40x10^{-2}ms^{-1}$. The tube is emptied and filled with another liquid at the same temperature and of density $900kgm^{-3}$. When the metal sphere falls centrally down the tube, it is found to attain a terminal velocity of magnitude $25x10^{-2}ms^{-1}$. Determine at 35^0C, the ratio of the coefficient of viscosity of the second liquid to that of the first. **[an 1.640]**

In an experiment to determine the coefficient of viscosity of motor oil, the following measurements were made

Mass of glass of sphere = $1.2x10^{-4}kg$

Diameter of sphere = $4.0x10^{-3}$,

Terminal velocity of sphere = $5.4x10^{-2}ms^{-1}$

Density of oil = $860kgm^{-3}$

Calculate the coefficient of viscosity of the oil [**an 0.45Nsm^{-2}**]

A metal sphere of radius $3.0x10^{-3}m$ and mass $4.0x10^{-4}kg$ falls under gravity, central down a wide tube filled with a liquid at 25 , the density of the liquid is $800kgm^{-3}$, the sphere attains a terminal velocity of magnitude . The tube is emptied and filled with another liquid at the same temperature and of density $100kgm^{-3}$. When the metal sphere falls centrally down the tube, it is found to attain a terminal velocity of magnitude . Determine at 25^0C, the ratio of the coefficient of viscosity of the second liquid to that of the first. **[An 2.09]**

A steel sphere of diameter $3.0x10^{-3}m$ falls through a cylinder containing a liquid x .When the sphere has attained a terminal velocity, it takes 1.08 s to travel between two fixed marks on the cylinder. When the experiment is repeated using another steel sphere of diameter $5.0x10^{-3}m$ with the cylinder containing liquid y, the time of fall between two fixed points is 4.8 s. if the density of liquid x is , that of liquid y is and that of the steel ball is determine the ratio of the coefficient of viscosity of the liquid x to that of the liquid y, if the temperature remains constant throughout. **[An 0.77]**

4. SURFACE TENSION

Surface tension is due to intermolecular attraction of the liquid molecules. The molecules in the surface are more spaced than those in the bulk of the liquid. So there is a net attractive force between them and this makes the liquid surface behave like an elastic skin in tension.

Q: What observations do suggest that the surface of a liquid acts like an elastic skin covering the liquid?

Surface tension, γ, of a liquid is defined as the force per metre acting in the surface at right angles to one side of a line drawn in the surface.

Thus, γ has dimensions $\dfrac{MLT^{-2}}{L} = MT^{-2}$

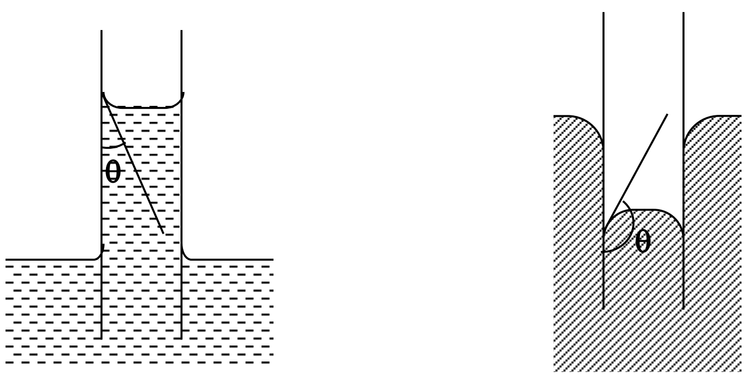
The unit is N m^{-1} (or kg s^{-2})

Why Large Mercury Drops Tend to Flatten but Small ones Assume a Spherical Shape

The ratio "Volume to surface area" of a sphere = $\frac{1}{3}$r, where r is the radius of the sphere. The weight pulls the centre of gravity down, tending to flatten the drop while surface tension tends to keep the drop in a spherical shape. From the above ratio it follows that the ratio "weight to surface tension" is proportional to $\frac{1}{3}$r and it increases with r. So the influence of weight is greater than that of the surface tension force for larger drops. So larger mercury drops tend to flatten.

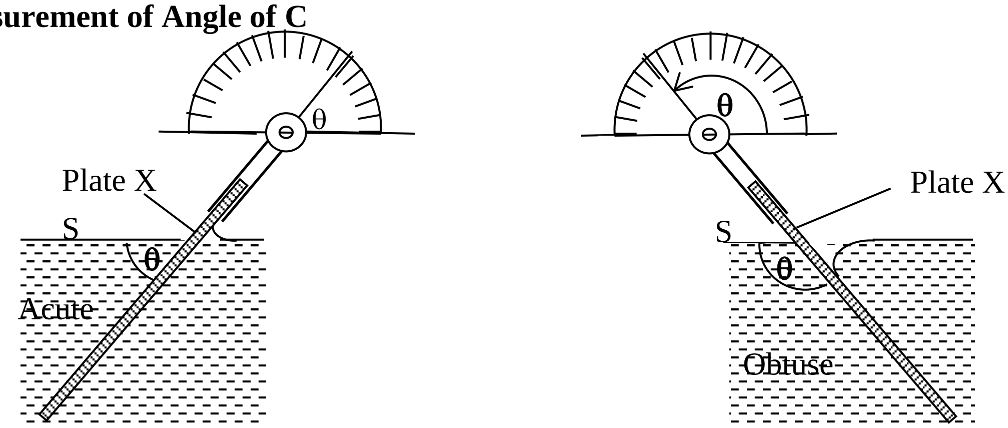
Angle of Contact

This is defined as ***the angle between the solid surface and the tangent plane to the liquid surface, measured through the liquid.*** See diagram.



Factors affecting the Angle of contact:
- Freshness of the liquid
- Cleanliness of the container

Measurement of Angle of C



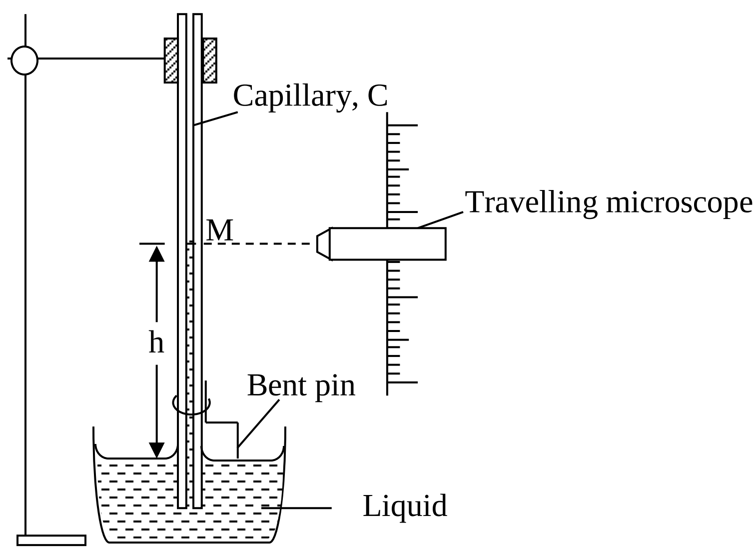
A plate, X, of the solid is placed at varying angles to the liquid until the surface, S, of the liquid appears to meet X without curving. Then the angle θ (through the liquid), between S and the plate, is measured. It is equal to the angle of contact.

Measurement of γ by Capillary Tube Method

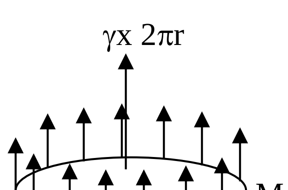
- A pin is bent at right angles at two places and fixed to the capillary tube, C, so that its tip just touches the liquid.
- The travelling microscope is focused at the meniscus, M, and the reading on the scale is noted.
- The beaker is removed and the microscope is focused at the tip of the pin and the scale reading is noted. Then the capillary rise, h, is calculated.
- The radius, r, of the capillary tube is obtained by cutting it at M and measuring using a travelling microscope. OR A length, *l*, and mass, m, of a mercury thread drawn in the tube are measured and r is calculated from

$$r = \sqrt{\frac{m}{\pi l \rho}}$$

whereρ = density of mercury



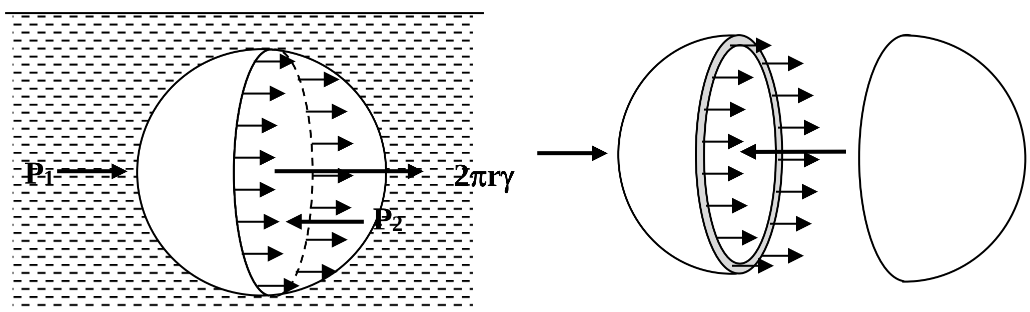
Theory:



The column of liquid of height h is held up by the surface tension force acting along the circumference of the meniscus.
Now, weight of the liquid column = $\pi r^2 h\rho g$
and the surface tension force = $2\pi r\gamma$ (for $\theta = 0$)
(or $2\pi r\gamma\cos\theta$)
Thus $2\pi r\gamma = \pi r^2 h\rho g$

$$\therefore \quad \gamma = \frac{rh\rho g}{2}\left(\text{or } \gamma = \frac{rh\rho g}{2\cos\theta}\right)$$

Pressure Difference in a Bubble (or Curved Surfaces)



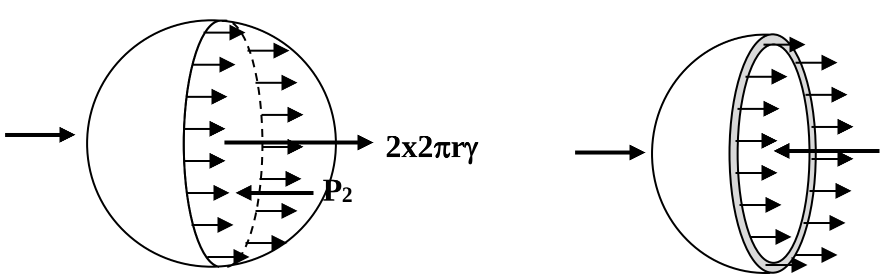
Consider a spherical bubble of radius r inside a liquid of surface tension γ
Let P_1 = pressure outside the bubble
P_2 = pressure inside the bubble
Taking one half of the bubble, its equilibrium is due to three forces, i.e.
Surface tension force + force due to outside pressure = force due to inside pressure
Thus, $2\pi r\gamma = \pi r^2 P_1 = \pi r^2 P_2$
$\therefore 2\gamma = r(P_2 - P_1)$

$$\therefore \quad \mathbf{P_2 - P_1 = \frac{2\gamma}{r}}$$

For curved surface of a liquid with angle of contact θ, excess pressure $= \dfrac{2\gamma\cos\theta}{r}$

Excess Pressure in a Soap Bubble

A soap bubble has two surfaces. So the surface tension force = 2 x $2\pi r\gamma = 4\pi r\gamma$

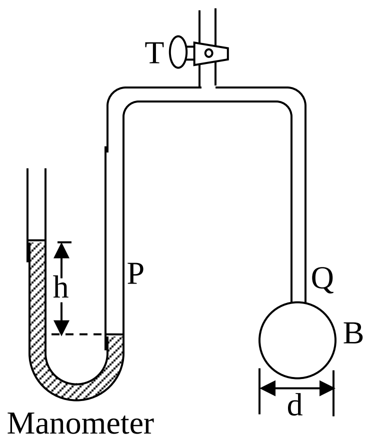


Thus, $4\pi r\gamma + \pi r^2 P_1 = \pi r^2 P_2$

$$\therefore \quad \mathbf{P_2 - P_1 = \frac{4\gamma}{r}}$$

Measurement of γof a Soap Bubble

- The apparatus is arranged as shown, with limb P connected to a manometer having a suitable liquid.



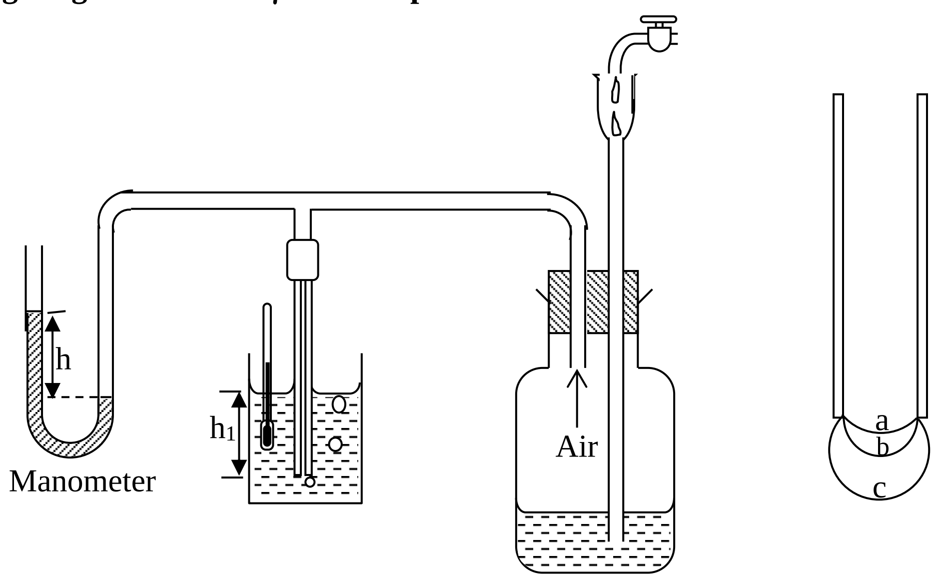
- With tap T open, the end of limb Q is dipped into the soap solution so that a little of it enters Q.
- T is closed and Q is withdrawn from the soap solution.
- T is opened again and, by gentle blowing through T, a bubble of reasonable size is formed at the end of Q and T is finally closed.
- The difference in manometer levels, h, and the diameter, d, of the bubble are each measured using a travelling microscope.

Now, excess pressure, $\Delta p = h\rho g$

$$\therefore \quad h\rho g = \frac{4\gamma}{r} = \frac{4\gamma}{d/2} = \frac{8\gamma}{d}$$

$$\therefore \quad \gamma = \frac{h\rho g d}{8}$$

Investigating Variation of γ with temperature



- The apparatus is set up as shown, with a capillary tube connected an air vessel and a manometer containing a suitable liquid. The lower end of the capillary tube is dipped into a beaker containing a liquid.
- The temperature of the liquid is read and noted.

- Water from a tap is slowly run into the air vessel while observing the manometer levels. The height h is observed to rise to maximum before collapsing. The maximum value of h is noted.
- The temperature of the liquid is changed and the procedure is repeated.

Now the manometer height is maximum when the diameter of the bubble in the liquid inside the air vessel is minimum. i.e. when the diameter of the bubble equals that of the capillary tube.
Let P = atmospheric pressure
ρ = density of liquid in the manometer
h_1 =depth of end of the capillary tube below the liquid surface
ρ_1 = density of the liquid in the beaker.
Then, inside pressure = $P + h\rho g$
Pressure outside the bubble = $P + h_1\rho_1 g$
Excess pressure = $(P + h\rho g) - (P + h_1\rho_1 g)$
= $(h\rho - h_1\rho_1)g$

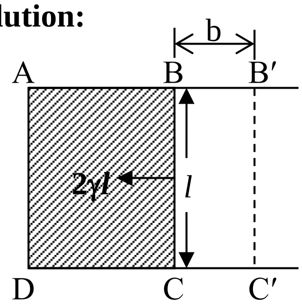
$$\therefore \quad \frac{2\gamma}{r} = (hp - h_1\rho_1)g$$

$\therefore \gamma = \frac{1}{2} rg(h\rho - h_1\rho_1)$
γ is found to be less at higher temperatures.

Examples:

1. Show that the surface tension of a liquid is equal to the work done per unit area in increasing the surface area of a liquid under isothermal conditions.

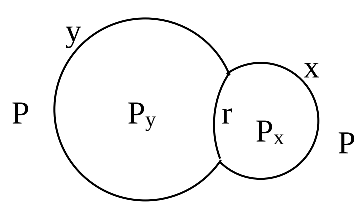
Solution:



Imagine a frame of wire BADC with a rod BC free to slide along AB and DC.
The arrangement encloses a liquid film ABCD, having two surfaces.
Suppose the rod BC moves to B′C′
The work done = $2\gamma l$ x b = γ x $2bl$
But $2bl$ is the total increase in surface area of the film
$\therefore \gamma$ = work done per unit area in enlarging the area.

2.Two soap bubbles, one of radius x and the other of larger radius y, get attached to each other. Find an expression for the radius of the common interface in terms of x and y.

Solution:



Let P = atmospheric pressure
P_x = pressure inside smaller bubble
P_y = pressure inside larger bubble

$$\text{Then} \quad P_x - P = \frac{4\gamma}{x} \quad \ldots\ldots\ldots\ldots\ldots..(1)$$

$$P_y - P = \frac{4\gamma}{y} \quad \ldots\ldots\ldots\ldots\ldots..(2)$$

$$P_x - P_y = \frac{4\gamma}{r} \quad \ldots\ldots\ldots\ldots\ldots..(3)$$

Eq(3) + eq(2) – eq(1) gives

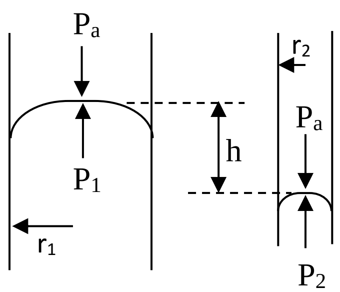
$$\frac{4\gamma}{r} + \frac{4\gamma}{y} - \frac{4\gamma}{x} = 0$$

$\therefore$ xy + rx – ry =0

$$\therefore \qquad \mathbf{r} = \frac{\mathbf{xy}}{\mathbf{y - x}}$$

3. Two tubes, one of radius 1.5 mm and the other 5 mm, stand vertically side by side with their lower ends submerged in mercury. If the angle of contact is 120°, find the difference in the levels of mercury in the two tubes. [Density of mercury = 13600 kgm^{-3}]

Solution:



$r_1 = 5 \times 10^{-3}$ m, $r_2 = 1.5 \times 10^{-3}$ m

Let P_a = atmospheric pressure

P_1 = pressure on the liquid side of the meniscus in the larger tube

P_2 = pressure on the liquid side of the meniscus in the smaller tube

h = difference in liquid levels

$$\text{Then } P_a - P_1 = \frac{2\gamma\cos\theta}{r_1} \quad \ldots\ldots\ldots(1)$$

$$\text{and } \quad P_a - P_2 = \frac{2\gamma\cos\theta}{r_2} \quad \ldots\ldots\ldots(2)$$

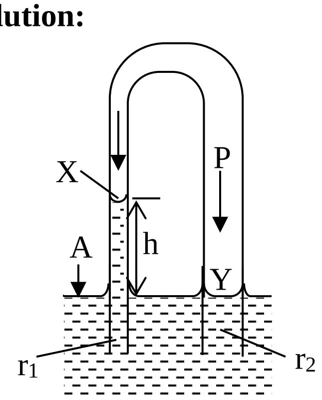
$$\text{Eq(1)} - \text{eq(2)}:\ P_2 - P_1 = h\rho g = 2\gamma\cos\theta\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\therefore \qquad h = \frac{2\gamma\cos\theta}{\rho g}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= \frac{2 \times 0.4 \times \cos 120^o}{13600 \times 9.81}\left(\frac{10^3}{5} - \frac{10^3}{1.5}\right) = \mathbf{1.4 \times 10^{-3}\ m}$$

4. A glass U-tube is inverted with the open ends of the straight limbs, of diameters respectively 0.50 mm and 1.00 mm, below the surface of water in a beaker. The U-tube is pushed down until the meniscus in one limb is level with the water outside. Find the height of water in the other limb. [Density of water = 1000 kg m^{-3}, $\gamma_{water} = 7.5 \times 10^{-2}$ N m^{-1}]

Solution:



r_1 = 0.025 cm

r_2 = 0,050 cm

Let Y be the meniscus level with the outside and h the height of meniscus X above Y.

$$\text{Then } P - (A - h\rho g) = \frac{2\gamma}{r_1} \quad \ldots.. \quad (1)$$

where A is the atmospheric pressure

$$\text{and for Y} \quad P - A = \frac{2\gamma}{r_2} \quad \ldots\ldots\ldots \quad (2)$$

$$\text{From (1)and (2)} \quad h\rho g = \frac{2\gamma}{r_1} - \frac{2\gamma}{r_2}$$

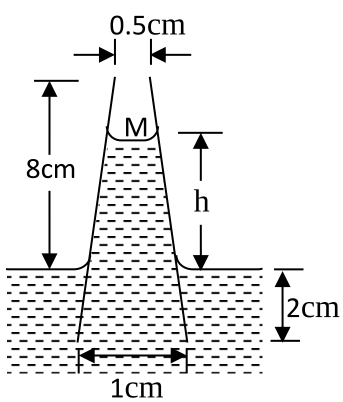
$$\therefore \quad h = \frac{2\gamma}{\rho g}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= \frac{2 \text{ x } 7.5 \text{ x } 10^{-2}}{1000 \text{ x } 9.8}\left(\frac{1}{0.25} + \frac{1}{0.5}\right)\frac{1}{10^{-3}} \quad = \quad 3.1 \text{ x } 10^{-2} \text{ m}$$

5. A vertical capillary tube 10 cm long tapers uniformly from an internal diameter of 1 mm at the lower end to 0.5 mm at the upper end. The lower is 2 cm below the surface of a liquid of surface tension 6 x 10^{-2} N m^{-1}, density 1200 kg m^{-3} and $\theta = 0^o$. Calculate the capillary rise, h.

Solution:

Suppose M is the meniscus at a height h.



The change in radius per cm height is

$$\frac{0.5 - 0.25}{10} = 0.0025 \text{ cm}$$

$\therefore$ At M the radius is r = [0.05 – 0.0025(h + 2)] x 10^{-2} m

= (0.045 – 0.0025h) x 10^{-2} m

The pressure above M is atmospheric, A

The pressure below M is A – hρg

$$\therefore \text{ pressure difference} = h \text{ x } 10^{-2}\rho g = \frac{2\gamma}{r}$$

$$= \frac{2 \text{ x } 6 \text{ x } 10^{-2}}{(0.045 - 0.0025h) \text{x } 10^{-2}}$$

$$\therefore \quad h\rho g \quad = \frac{200 \text{ x } 6 \text{ x } 10^3}{45 - 2.5h}$$

$$\therefore \quad 45h - 2.5h^2 \quad = \frac{200 \text{ x } 6 \text{ x } 10^3}{1200 \text{ x } 9.8}$$

$\therefore$ $2.5h^2 - 45h + 102 = 0$

$\therefore$ h = 2.66 cm

Exercise:

1. A soap bubble has diameter of 4 mm. Calculate the pressure inside it if the atmospheric pressure is 10^5 N m^{-1}. [γfor soap solution is 2.8 x 10^{-2} N m^{-1}]

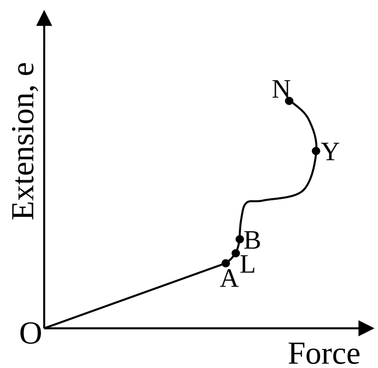
2. Estimate the total surface energy of a million drops of water each of radius 10^{-4}m, if surface tension for water is 7 x 10^{-2} N m^{-1}. State the assumptions made.

1. Calculate the change in surface energy of a soap bubble when its radius decreases from 5 cm to 1 cm. [γ = 2 x 10^{-2} N m^{-1}]

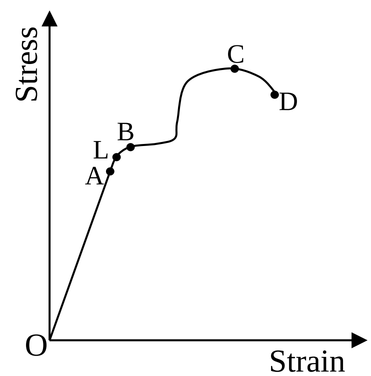
5. ELASTICITY

A material is said to be elastic if when it is deformed by a force and then released it regains its original dimensions.

The graphs below show:
(i) extension against load and (ii) stress against strain
for a wire of ductile material.



(i)



(ii)

A = ***Proportional Limit*****:** In the region OA the extension is proportional to the load.

L = ***Elastic Limit***: This is the maximum load a body can experience and still regain its original dimensions when released. In the region AL the wire returns to its original dimensions when unloaded, but the extension is no longer proportional to the load.

B = ***Yield Point***: At this value of the load, the molecules of the wire begin to slide across each other, so that the material becomes ***plastic***.

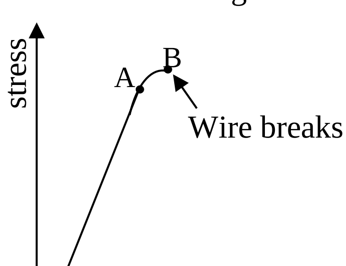
C = ***Breaking Stress***: (Ultimate stress). It is the maximum stress a material can withstand without snapping.

Hooke's Law

Provided the proportional limit is not exceeded the extension is proportional to the tension in the wire.

Ductile and Brittle Materials

A substance is said to be ***ductile*** if it undergoes considerable plastic deformation before breaking, e.g. copper.
Brittle materials break just after the elastic limit is reached.
Below is a curve of stress against strain for a brittle material.



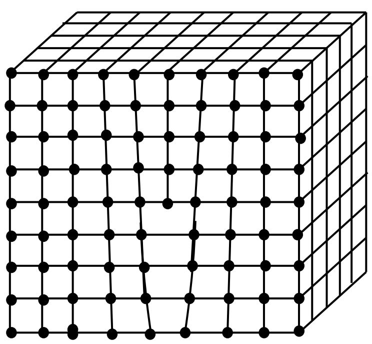
Structure and Behaviour of Materials

Metals have a crystalline structure in which the atoms are arranged in a regular, repetitive manner forming a 3-dimensional lattice.
Other solids, e.g. glass, have no ordered structure and are said to be ***amorphous***.
Crystalline structures may have imperfections due to:

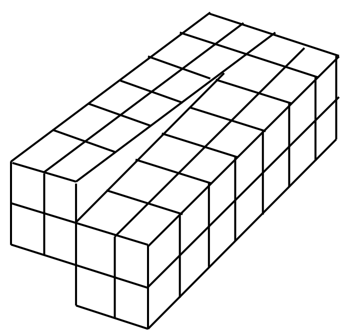
(i) presence of foreign atoms, e.g. carbon in steel
(ii) existence of vacancies
(iii) regions of disorder in the structure

Dislocations and Mechanical Behaviour

A dislocation is a region of disorder within an otherwise ordered structure. It is the most important defect. See the figures below.



(i) Edge dislocation



(ii) Screw dislocation

Ductile materials have many dislocations and these dislocations are free to move within the structure. So the material undergoes considerable plastic deformation.
In brittle materials, e.g. glass, there are no dislocations
Toughness is the ability to resist crack growth. This is brought about by the ability of dislocations to move and blunt the crack tip.
Hardness is the resistance to plastic deformation

Work Hardening

Work hardening is the increase in hardness of a substance as a result of the substance being subjected to repeated cycles of plastic deformation. This is because under repeated stress, the

dislocations move and intersect. They get entangled (pinned) and so become immobile. So the substance hardens and becomes brittle.
Annealing helps to restore the ductile state. This is the process of heating a metal to a high temperature below its melting point and keeping it at that temperature for some time.

Tensile Stress and Strain

When a force, F, is applied to the end of a wire of cross-sectional area A, then

Tensile stress = force per m^2 of its cross-sectional area = $\frac{F}{A}$

If the extension of the wire is e, and its original length is *l*, then

Tensile strain = extension per metre length of the wire = $\frac{e}{l}$

The ***modulus of elasticity*** (E) is defined as the ratio

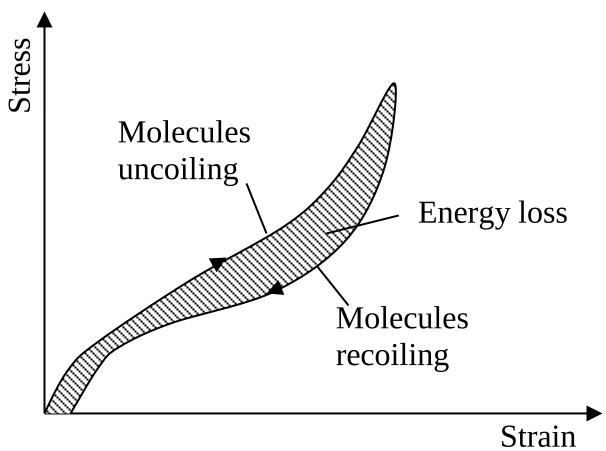
$$\mathrm{E} = \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F/A}{e/l}$$

Hence the dimensions of E are $\frac{MLT^{-2}L^{-2}}{LL^{-1}} = ML^{-1}T^{-2}$

The unit is N m^{-2}.

Behaviour of Rubber

Rubber molecules are coiled. When a stress is applied, the material elongates considerably because the molecules are uncoiling. Below is a graph of stress against strain, which indicates a ***hysteresis loop***.

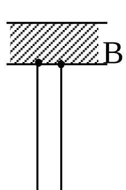


On unloading, the molecules do not recoil in the same way as they were before. So,a hysteresis loop results. Hence when rubber is stretched and then released, some energy is lost in form of heat (represented by the shaded area).

When rubber is heated, the tendency for its molecules to coil up increases and the molecules become more difficult to uncoil. So:

(i) the material shortens, and
(ii) its Young's modulus increase

Determination of Young's Modulus



Two thin, long wires P,Q of the material whose Young's modulus is required are supported at B. The use of the two wires P and Q helps to eliminate the correction for the yielding of the support and for the changes in temperature. P is kept taut by a weight R attached to its lower end and caries a scale a millimetre scale, M. Q carries a vernier scale, V, which is alongside the scale M.

-The original length, l, of Q is measured from the top B to the vernier.

-The diameter of Q is measured at several places and the average value, d, calculated, from which the cross-sectional area, A = $\frac{1}{4}\pi d^2$, is found.

-Loads are added in steps of 1 kg each time determining the corresponding extension of the wire.

-The extensions are also determined as the load is gradually removed in the same steps – they should be very nearly the same if the elastic limit was not exceeded.

A graph of extension, e, against the load (in N) is plotted.

It is a straight line with gradient $= \dfrac{l}{EA}$

where E = Young's modulus, since e = $\dfrac{l}{EA}.F$

Thus, $$E = \frac{l}{\text{A x gradient}}$$

Force in a Bar due to Contraction or Expansion

Consider a bar of Young's modulus, E, cross-sectional area A and linear expansivity α. Suppose the temperature of the bar is lowered by θ^oC. Then, if the original length of the bar is l, the decrease in length, e, if the bar were free to contract, is $\alpha l\theta$.

Now $$E = \frac{F/A}{e/l}$$

$$\therefore \quad F = \frac{EAe}{l} = \frac{EA\alpha l\theta}{l} = \mathbf{EA\alpha\theta}$$

This is the force set up in the bar if it is not free to contract.

Energy Stored in a Wire

If E = force in the wire at an extension x, then $F = \dfrac{EAx}{l}$

The work done in extending the wire by a small distance dx is

$$dW = F.dx = \frac{EAx}{l}.dx$$

The total work done in extending the wire by e is

$$W = \frac{EA}{l}\int_0^e x dx$$

$$\therefore W = \frac{EAe^2}{2l} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

This is the energy stored in the wire when it is extended by e.

Now, the volume of the wire = Al

Thus, energy stored per unit volume $= \frac{EAe.e}{2l.Al} = \frac{1}{2}.\frac{Fe}{Al} = \frac{1}{2}.\frac{F}{A}.\frac{e}{l}$

$= \frac{1}{2}$ **x stress x strain**

OR Energy per unit volume= $\frac{EAe.e}{2l.Al} = \frac{1}{2} \text{ x } E \text{ x} \left(\frac{e}{l}\right)^2$

$= \frac{1}{2}$ **xE x (strain)^2**

Examples:

A copper wire is fused at one end to an iron wire. The copper wire has length 90 cm and cross-section 9 x 10^{-3} cm^2. The iron wire has length 140 cm and cross-section 1.3 x 10^{-2} cm^2. The compound wire is stretched and its total length increases by 1.0 cm. Calculate

(i) the ratio of the extensions of the two wires
(ii) the extension of each wire
(iii) the tension applied to the compound wire

[E_{copper} = 1.30 x 10^{11}Pa; E_{iron} = 2.10 x 10^{11}Pa]

Solution:

(i) The wires experience the same force

$$\therefore \quad \frac{E_c A_c e_c}{l_c} = \frac{E_i A_i e_i}{l_i}$$

$$\therefore \quad \frac{e_c}{e_i} = \frac{E_i A_i l_c}{E_c A_c l_i} = \frac{2.1 \text{ x } 1.3 \text{ x } 90}{1.3 \text{ x } 0.9 \text{ x } 140} = \frac{3}{2}$$

(ii) $e_c + e_i = 1.0$

$\therefore \quad (1 + \frac{2}{3})e_c = 1$

$\therefore e_c = 1 \text{ x } \frac{3}{5} =$ **0.6 cm**

$\therefore e_i =$**0.4 cm**

(iii) Tension, $F = \frac{E_c A_c e_c}{l_c}$

$$= \frac{1.3 \text{ x} 10^{11} \text{ x } 9 \text{ x } 10^{-7} \text{ x } 0.6}{90} = \mathbf{780 \text{ N}}$$

CHAPTER 6: STATICS

Is a subject which deals with equilibrium of forces the forces which act on a bridge.

Coplanar forces

Those are forces acting on the same point (plane).

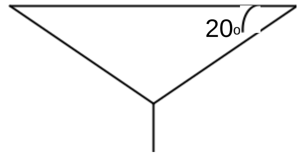
Conditions necessary for mechanical equilibrium When forces act on a body then it will be in equilibrium when;

the algebraic sum of all forces on a body in any direction is zero
the algebraic sum of moments of all forces about any point is zero

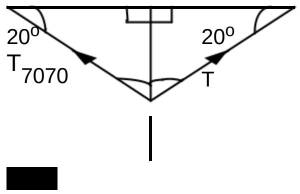
Examples

A mass of 20kg is hang from the midpoint P of a wire as shown below. Calculate the tension in the wire take g=9.8ms-1

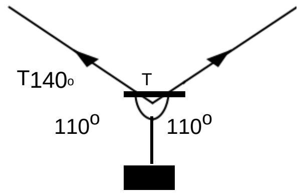
20°



Solution



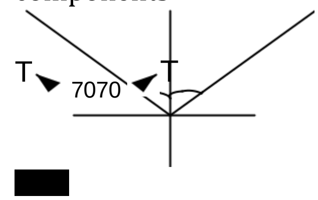
Method I
lami's theorem
(Apply to only three forces in equilibrium)



T=

T= 286.83N

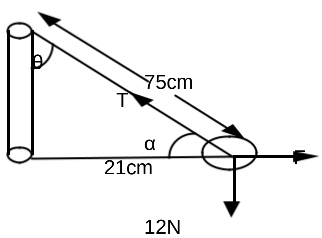
METHOD II using components



Resolving vertically
Tsin70 +Tcos70 = 20gN
2Tcos70 = 20x9.81
T
T 286.83N

2. One end of a light in extensible string of length 75cm is fixed to a point on a vertical pole. A particle of weight 12N is attached to the other end of the string. The particle is held 21cm away from the pole by a horizontal force. Find the magnitude of the force and the tenion in the string

Solution

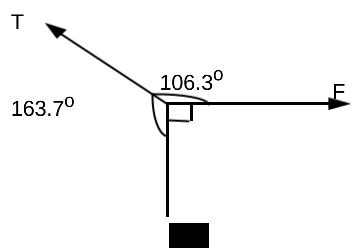


Sin =

Also

= 73.7°

Using Lami's theorem

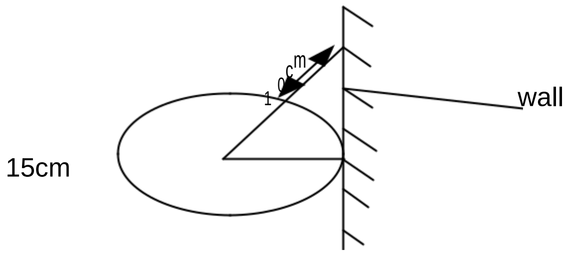


F = 3.51N

Also

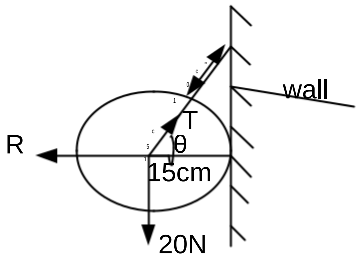
T = 12.5N

A sphere of weight 20N and radius 15cm rests against a smooth vertical wall. A sphere is supported in its position by a string of length 10cm attached to a point on the sphere and to a point on the wall as shown.

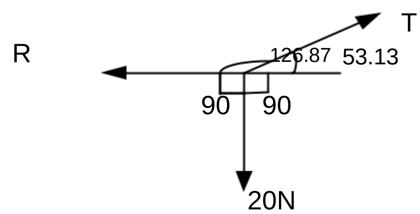


i) copy the diagram and show the forces acting on the sphere
ii) Calculate the reaction on the sphere due to the wall.
iii) Find the tension in the string

Solution

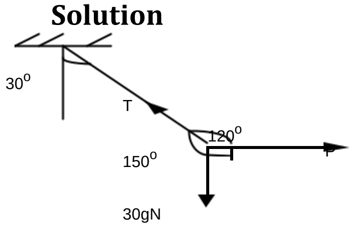


Using Lami's theory



4. A mass of 30Kg hangs vertically at the end of a light string. If the mass is pulled aside by a horizontal force P so that the string makes an angle 30^0 with the vertical. Find the magnitude of the force P and the tension in the string.

Solution



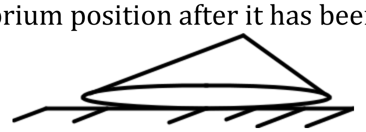
P = 169.91N

T= 339.83N

: Types of equilibrium

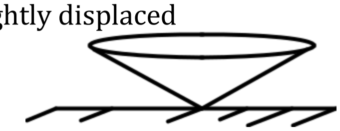
Stable equilibrium.

This when a body returns to its equilibrium position after it has been slightly displaced



Unstable equilibrium.

This is when a body does not return to its equilibrium position and does not remain in the displaced position after it has been slightly displaced



Neutral equilibrium.

This is when a body stays in the displaced position after it has been slightly displaced

Turning effect of forces

A force can produce a turning effect or moment about a pivot, this can be a clockwise or anti clockwise turning effect.

Moment of a force

This is the product of a force and the perpendicular distance of its line of action from the pivot. The unit of a moment is Nm and it's a vector quantity.

Moment of a force Force x perpendicular distance of its line of action from pivot.

Principle `of moments

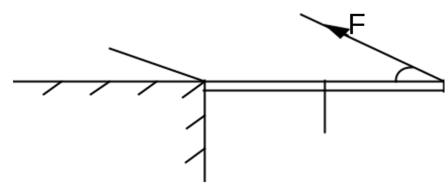
It states that when a body is in equilibrium, the sum of clockwise moments about a point is equal to the sum of anticlockwise moments about the same point.

Examples

A hinged trapped door of mass 15kg and length 1m is to be opened by applying a force F at an angle of 45^0 as shown below.

Hinge

45^o



Calculate

The value of F

The horizontal force on the hinge

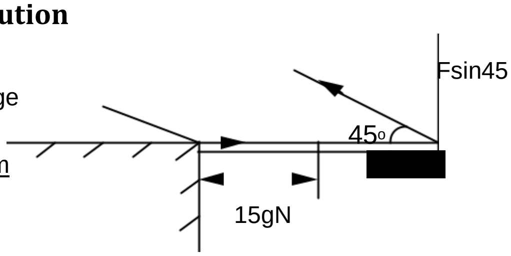
Solution

F

Hinge

R

0.5m



Taking moments about the hinge At equilibrium,

Anti clockwise moments = clockwise

√

F 104.1N

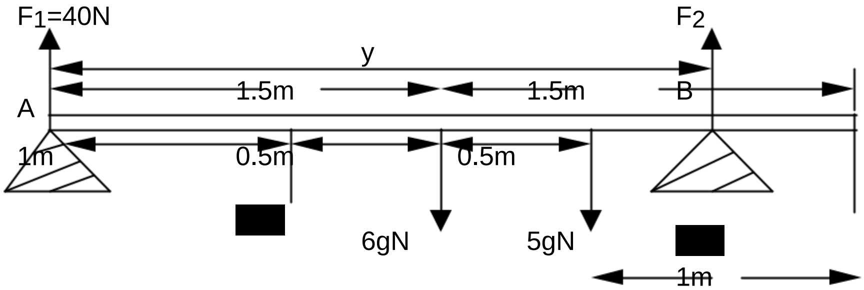
The horizontal force is the horizontal normal reaction

Resolving horizontally:

2. A uniform bean AB, 3m long and of mass 6kg is supported at A and at another point.

A load of 1 kg is suspended at B, loads 4kg and 5 kg at points 1m and 2m from A. If the pressure on the support at A is 40N. Where is the other support?

Solution



Note.
Uniform beam implies its weight acts at the centre of gravity (mid-point)
Resolving vertically:

40 +F_2 = 4x9.81 + 6x9.81+ 5x9.81 + 1x9.81

Taking moments about A at equilibrium

Clockwise moment = anti clockwise moment
4gNx1 +6gNx1.5 +5gNx2+1gNx3 = x y

Y

Y

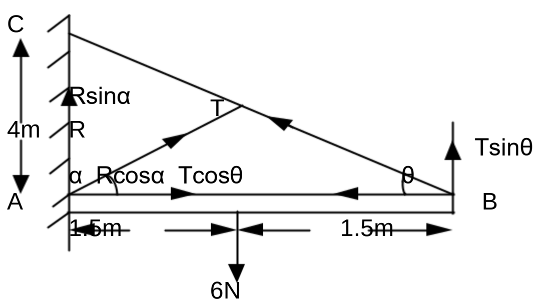
Y = 2.18m

The other support is 2.18 from A

Beams hinged against the wall

1. A Uniform beam AB, 3.0m long and of weight 6N is hinged at a wall at A and is held stationary by a rope attached to B and joined to a point C on the wall, 4.0m vertically above A. Find the tension T in the rope and the Reaction R of the hinge.

Solution



tan – = 53.13^0

Taking moments about A at equilibrium
Clockwise moment = anticlockwise moment
( )

Resolving vertically:

------------------------------i
Resolving horizontally:

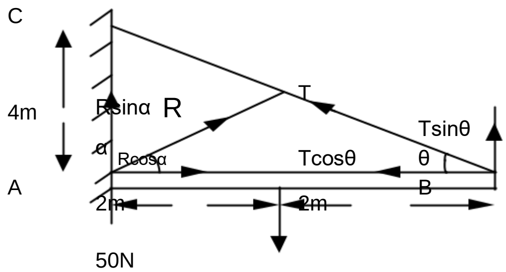
-----------------------------ii
i/ii
=

tan

53.28^0
Put into i
Rsin = 3
Rsin53.28 =3
R = 3.74N
The reaction at A is 3.74 at 53.28^0 to the beam

A uniform beam AB of length 4m and weight 50N is freely hinged at A to a vertical wall and is held horizontal in equilibrium by a string which has one end attached at B and the other end attached to appoint C on the wall, 4m above A. find the magnitude of the reaction at A.

Solution



tan − = 45

Taking moments about A

R = 35.36N at 45^0 to the beam

Resolving vertically: =

…………………….(i)

Resolving horizontally:

…………………….(ii)

(i)/(ii)

tan = = 45^0

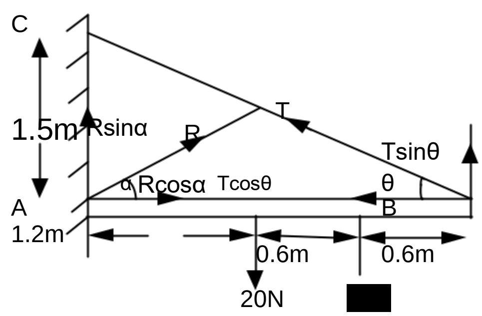
Put into (ii)

A uniform beam AB of mass 20kg and length 2.4m is hinged at a point A in a vertical wall and is maintained in a horizontal position by means of a chain attached to B and to point C in a wall 1.5m above. If the bar carries a load of 10kg at a point 1.8m from A. calculate.

The tension in the chain

The magnitude and direction of the reaction between the bar and the wall

Solution



tan — = 32.01

Taking moments about A

=

Tension in the chain

(ii) Reaction at the wall

Revolving vertically

R sin + Tsin = 20gN + 10gN

R sin + 323.87sin32.01 = 30gN

R sin = 122.63……………………(i)

Resolving horizontally; = Tcos

32.87cos32.01

274.63 ………………(ii)

(i)/(ii)

=

tan = 0.446528055

= 24.1^0 Put in eqn (ii)

R cos24.1 = 274.63

R = 300.85N

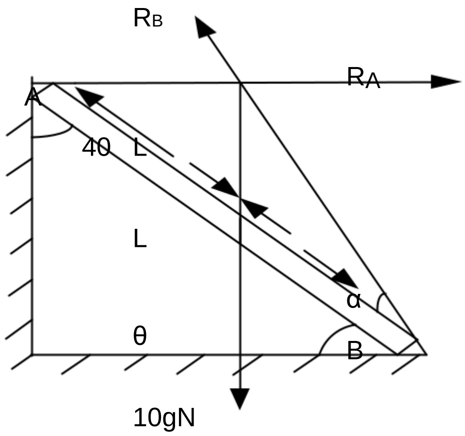
Reaction at A is 300.85 at 24.1^0 to the horizontal

Ladder problems

A uniform rod AB of mass 10kg is smoothly hinged at B and rests in a vertical plane with the end A against a smooth vertical wall. If the rod makes an angle of 40^0 with the wall, find the reaction on the wall and the magnitude of the reaction at B

Solution

Note: If there are three coplanar forces acting on a rigid body which is equilibrium, then their line of actioN meet at some specific point provided no force is parallel to each other.



Let the length of the ladder be 2L

+40₀+90₀ = 180₀

= 50₀

Taking moments about B

RA 2Lsin = 10gN

RA 2Lsin50 =

RA = 41.16N

Resolving vertically; RB sin(+) = 10gN

( ) ....(i)

Resolving horizontally: RB (+)=RA

RB ( ) ....(ii)

(i)/(ii)

( ) =

+ 67.24

+50 67.24

17.24₀

Put into equation (i)

RB ( ) ....(i)

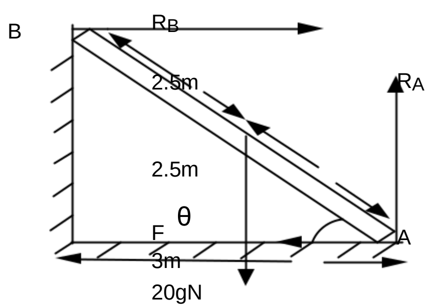
RB ( )

RB = 106.38N

Reaction at B is 106.38N at 67.24₀ to the horizontal.

A uniform ladder which is 5m long and has a mass of 20kg leans with its upper end against a smooth vertical wall and its lower end on a rough ground. The bottom of the ladder is 3m from the wall. Calculate the functional force between the ladder and the ground and the coefficient of friction

Solution



= 53.13₀

Resolving vertically: RA = 20gN

RA= 20x9.81

RA = 196.2N

Taking moments about A

RB

RB 5sin53.13=20x9.81x2.5cos53.13

RB= 73.56N

Resolving horizontally: RB= F

F = 73.56N

But F = xRA

73.56 =

= 0.37

Exercise

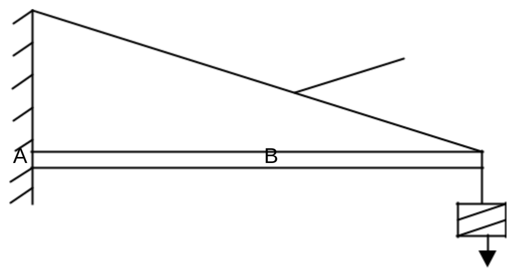
One end of a uniform plank of length 4m and weight 100N is hinged to the vertical wall. An inelastic rope, tied to the other end of the plank is fixed at a point 4m above the hinge. Find

The tension in the rope’

The reaction of the wall on the plank **An(388.9N, 302.1N at 24.4^0 to horizontal)**

2.

Cable of length 5m



300N

The figure shows a uniform rod AB of weight 200N and length 4m, the beam is hinged to the wall at A.

Find the tension in the cable

The horizontal and vertical components of the force exerted on the beam by the wall

The reaction of the wall on the beam at point A

An(666.7N, 533.3N, 99.98N, 542.59 at 10.6^0 to the horizontal)

A uniform beam AB of length 2L rests with end A in contact with a rough horizontal ground. A point C on the beam rests against a smooth support. AC is of length with C higher than A and AC making an angle of 60^0 with the horizontal. If the beam is in limiting equilibrium, find the coefficient of friction between the beam and the ground.

A uniform ladder of mass 25kg rests in equilibrium with its base on a rough horizontal floor and its top against a smooth vertical wall. If the ladder makes an angle of 75^0 with the horizontal, find the magnitude of the normal reaction and of the frictional force at the floor and state the minimum possible value of the coefficient of friction between the ladder and the floor.

A ladder 12m long and weighing 200N is placed 60^0 to the horizontal with one end B leaning against the smooth wall and the other end A on the ground. Find;

a) reaction at the wall **An(57.7N)**

reaction at the ground **An(208.2N at 73.9^0 to the horizontal).**

Couples

A couple is a pair of **equal**, **parallel** and **opposite** forces with different lines of action acting on a body.

A true couple produces only a **rotation** but not a **translation**

Examples

- Forces in the driver's hands applied to a steering wheel
- Forces in the handles of a bike
- Forces in the peddles of a bike
- Forces experienced by two sides of a suspended rectangular coil carrying current in a magnetic field.

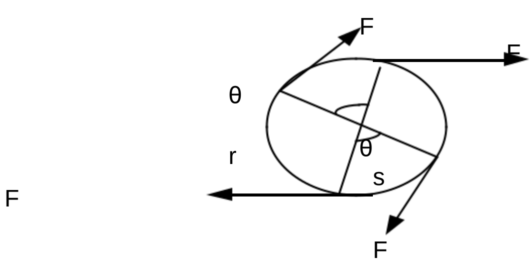
Moment of a couple (torque of a couple)

It is defined as the product of one of the forces and the far distance between the lines of action of the forces

Moment of a couple or torque of couple = F x d

Work done by a couple

Consider two opposite and equal forces acting tangentially on a wheel of radius r, suppose the wheel rotates through an angle radians as shown below.



Work done by each force

But

$360^0 = 2\pi$

Work done by each force

Total work done by the couple

CENTRE OF MASS

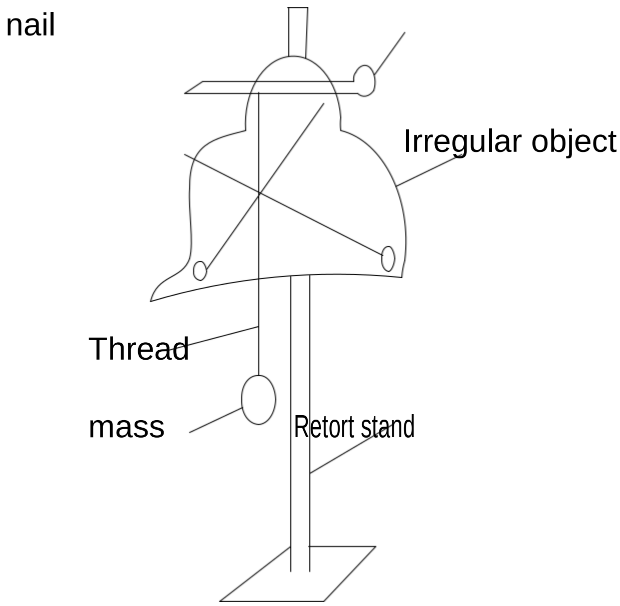
This is a point where an applied force produces a linear acceleration but not an angular acceleration (rotation)

CENTER OF GRAVITY

This point where the entire weight or resultant force of attraction of the body acts.

DETERMINATION OF CENTRE OF GRAVITY OF AN IRREGULAR LAMINA

Procedures:



Clamp a nail on the stand so that the pointed end is free.

Make holes at three points at the edge of the card board and hung the card board on the nail through one of the hole.

Tie the thread on a mass to make a plumb line.

Tie the plumbline on the nail allow it to rest freely with its thread tacking the card board

Trace the thread using a pencils I.

Repeat the procedure when the plumbline is suspended from the other holes.

The point of intersection of the three lines is the centre of gravity of the board

UNEB 2009 No 2

Define the following terms

Velocity (2marks)

Moment of a force

c)(i) State the condition necessary for mechanical equilibrium t be attained.(2 marks)

ii)A uniform ladder of mass 40kg and length 5m, rest with its upper end against a smooth vertical wall and with its lower end at 3m from the wall on a rough ground. Find the magnitude and direction of the force exerted at the bottom of the ladder (06 marks)

An[418.7N at an angle of 69.4_0 to the horizontal]

UNEB 2006 No 2

State the condition for equilibrium of a rigid body under the action of coplanar forces. (2mk)

A 3m long ladder at an angle 60_0 to the horizontal against a smooth vertical wall on a rough ground. The ladder weighs 5kg and its centre of gravity is one third from the bottom of the ladder.

Draw a sketch diagram to show the forces acting on the ladder. (2mk)

Find the reaction of the ground on the ladder. (4mk)

(Hint Reaction on the ladder =√ **) An(49.95N at 79.11_0 to the horizontal)**

UNEB 2006 No1

e) Describe an experiment to determine the centre of gravity of a plane sheet of material having an irregular shape. (4 marks)

UNEB 2005 No2

f) (i) Define centre of gravity (1 mark)

Describe an experiment to find the centre of gravity of a flat irregular piece of a card board. (3 marks)

UNEB 2002 No2

d) (i)Define moment of a force (1 mark)

A wheel of radius 0.6m is pivoted at its centre. A tangential force of 4.0N acts on the wheel so that the wheel rotates with uniform velocity find the work done by the force to turn the wheel through 10 resolutions.

Solution

Work done =force x distances

But distance= circumference x number of revolutions

=2 rx10

W=F x d

=4x2 rx0.6x10

W =150.79J

UNEB 2000 No3

State the conditions for equilibrium of a rigid body under the action of coplanar forces.(2mk)

A mass of 5.0kg is suspended from the end A of a uniform beam of mass 1.0kg and length 1.0m. The end B of the beam is hinged in a wall. The beam is kept horizontal by a rope attached to A and to a point C in the wall at a height 0.75m above B

i. Draw a diagram to show the forces on the beam (2 marks)

ii. Calculate the tension in the rope (4 marks)

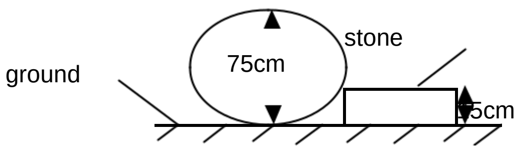
iii. What is the reaction exerted by the hinge on the beam (5 marks)

An (89.8N, 72.01N, at 3.95$_0$ to the beam)

UNEB 1998 No1

d) (i)Explain the term unstable equilibrium (3mk)

(ii) An oil drum of diameter 75cm and mass 90kg rests against a stone as shown



Find the least horizontal force applied through the centre of the drum, which will cause the drum to roll up the stone of height 15cm. **An(1177.2N)** (5 marks)